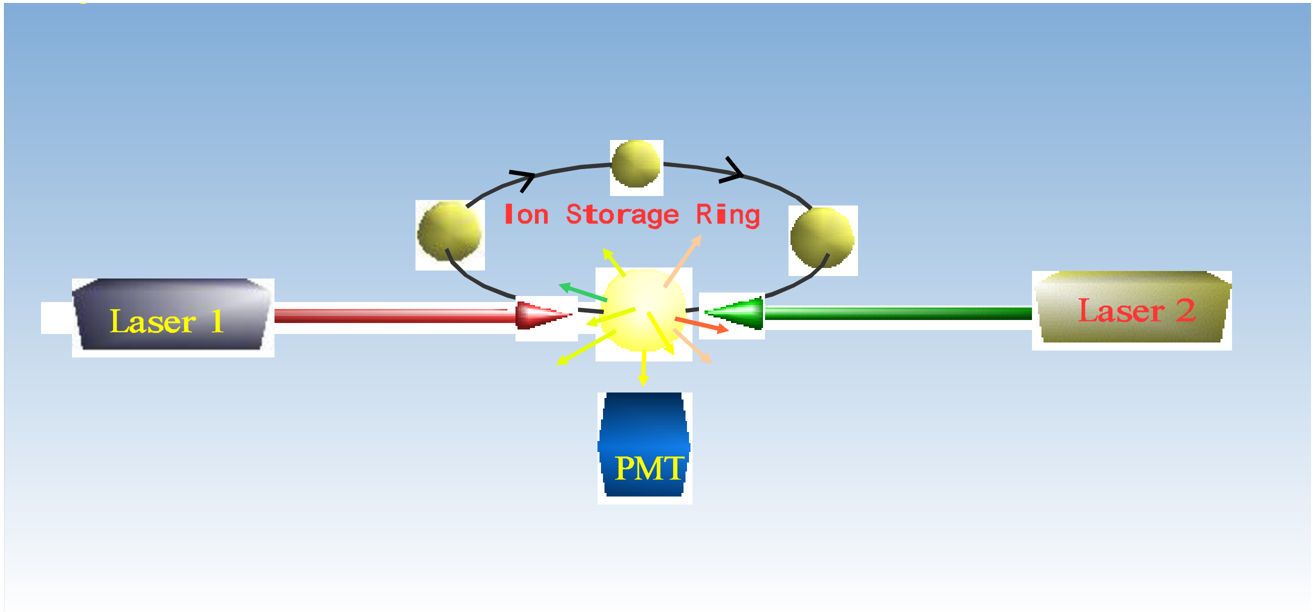
Test of Time Dilation by Laser Spectroscopy on Fast Ions



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A Test Theory of Special Relativity
Robertson (1949), Mansouri & Sexl (1977)

Assumptions:

- preferred ("ether") system $\Sigma(T, \vec{X})$
- speed of light c_0 isotropic in Σ
- lab. system $S(t, \vec{x})$ moving at velocity $\vec{w}c_0$ w.r.t. Σ (along X)
- assume linear transformation

Generalized Lorentz Transformation:

$$
\begin{aligned}
T &= \frac{1}{a}\left(t + \frac{wx}{c_0}\right) \\
X &= \frac{x}{b} + \frac{wc_0}{a}\left(t + \frac{wx}{c_0}\right) \\
Y &= \frac{y}{d}, \qquad Z = \frac{z}{d}
\end{aligned}
$$

SR: $a = 1/b = \sqrt{1 - w^2}, d = 1$

Low-velocity limit:

$a(w^2) = (1 - w^2)^{1/2}(1 + \delta\hat{\alpha} \cdot w^2 + \ldots)$	Time Dilation
$b(w^2) = (1 - w^2)^{-1/2}(1 + \delta\hat{\beta} \cdot w^2 + \ldots)$	Longitudinal Length Contraction
$d(w^2) = (1 + \hat{\delta} \cdot w^2 + \ldots)$	Transverse Length Contraction

$$\gamma_{\mathrm{MS}} \approx (1 - w)^{-1/2 - \delta\hat{\alpha}}$$

Three Parameters – Three experiments

Speed of light $c(\theta,w^2)$ in the moving frame S within the Mansouri-Sexl theory ($\theta=\sphericalangle(c,w)$):

$$\frac{c(\theta,w)}{c_0} = 1 + (\delta\hat{\beta} - \hat{\delta})w^2 \sin^2(\theta) + (\delta\hat{\alpha} - \delta\hat{\beta})w^2$$

- **Michelson-Morley (1887) experiments** $\rightarrow$ **Isotropy of c:**

  [P.Antonini et al., PRA (2005)] $\left|\delta\hat{\beta} - \hat{\delta}\right| \leq 3.1 \cdot 10^{-10}$

- **Kennedy-Thorndike (1932) experiments** $\rightarrow$ **Velocity-independence of c:**

  [P.Wolf et al., Gen. Rel. and Grav. (2004)] $\left|\delta\hat{\alpha} - \delta\hat{\beta}\right| \leq 3.0 \cdot 10^{-7}$

- **Ives-Stilwell (1938) experiments** $\rightarrow$ **Time dilation:**

  [G.Saathoff et al., PRL (2003)] $\left|\delta\hat{\alpha}\right| \leq 2.2 \cdot 10^{-7}$

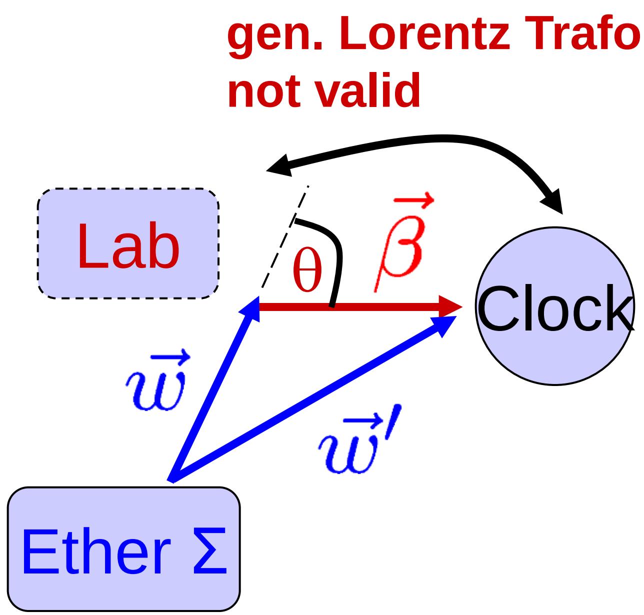
This Talk

Time Dilation in the Mansouri-Sexl Framework

Generalized time dilation for a clock moving at a velocity $\vec{w}'$ w.r.t. the ether

$$\gamma_{MS} = a^{-1} = \gamma_{SR}\left(1 + \delta\hat{\alpha} \cdot w'^2 + \ldots\right)$$

$$\vec{w}' \approx \vec{w} + \vec{\beta}$$



$\vec{w} = 350\ \mathrm{km/s}$ **if Σ = Cosmic microwave background rest-frame**

$$\gamma_{MS} = \gamma_{SR}\left(1 + \delta\hat{\alpha} \cdot \left(\beta^2 + 2\vec{\beta} \cdot \vec{w}\right) + \ldots\right)$$

Experimental Situation

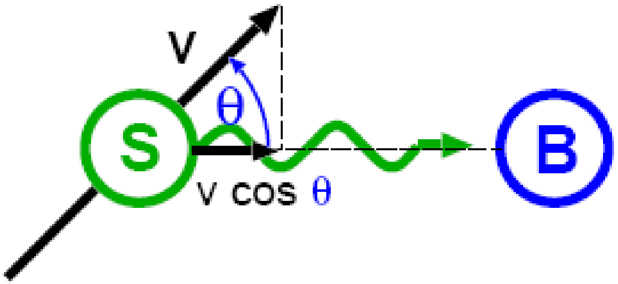
Typically two types of measurements:

- slow particles and high accuracy: use $\vec{\beta}\vec{w}$ term looking for sidereal variations
- spectroscopy with fast particles is sensitive to β^2 term

Experiment	limit on $\delta\hat{\alpha}$
Ives and Stilwell (1938)	1×10^{-2}
relativistic H^- beam (1986)	1.9×10^{-4}
Mößbauer rotor (1963)	1×10^{-5}
Gravity Probe A (1980)	2.1×10^{-6}
Two-photon transition in neon (1988)	1.4×10^{-6}
Two-photon transition in neon (1993)	2.3×10^{-6}
TSR Lambda-spectroscopy (1994)	8×10^{-7}

Measuring Time Dilation via the Optical Doppler Effect

The relativistic Doppler Effect:



$$\nu = \nu_0 \frac{1}{\gamma\,(1 - \beta \cos\theta)} \qquad \gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

- **Einstein (1907) :** $\theta = 90^\circ$ **(transverse Doppler effect)**

$$\longrightarrow \nu/\nu_0 = 1/\gamma$$

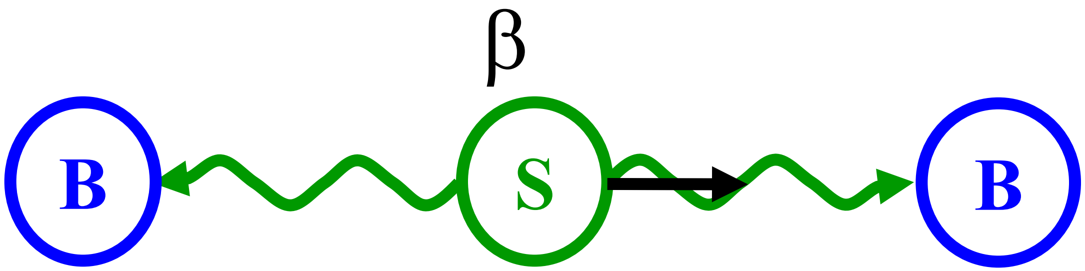
- **Advantage: independence of β**

- **Disadvantage:**
  - **independence of β**
  - **sensitivity to misalignment (cos θ ~ linear @ π/2)**

Idea by Ives and Stilwell 1938

measure with <u>and</u> against the particle motion $\theta = 0°$ and $\theta = 180°$

$$\nu_{\mathrm{p}} = \nu_0/\gamma(1-\beta)$$
$$\nu_{\mathrm{a}} = \nu_0/\gamma(1+\beta)$$



→ γ_{exp} **and** β **measured simultaneously**

Special Relativity:

$$\gamma \longrightarrow \gamma_{\mathrm{SR}} = (1-\beta^2)^{-1/2}$$

$$\Longrightarrow \nu_{\mathrm{p}}\nu_{\mathrm{a}} = \nu_0^2$$

Mansouri-Sexl Test Theory:

$$\gamma \longrightarrow \gamma_{\mathrm{MS}} = \gamma_{\mathrm{SR}}(1 + \delta\hat{\alpha}\cdot(\beta^2 + 2\vec{\beta}\vec{w}) + \ldots).$$

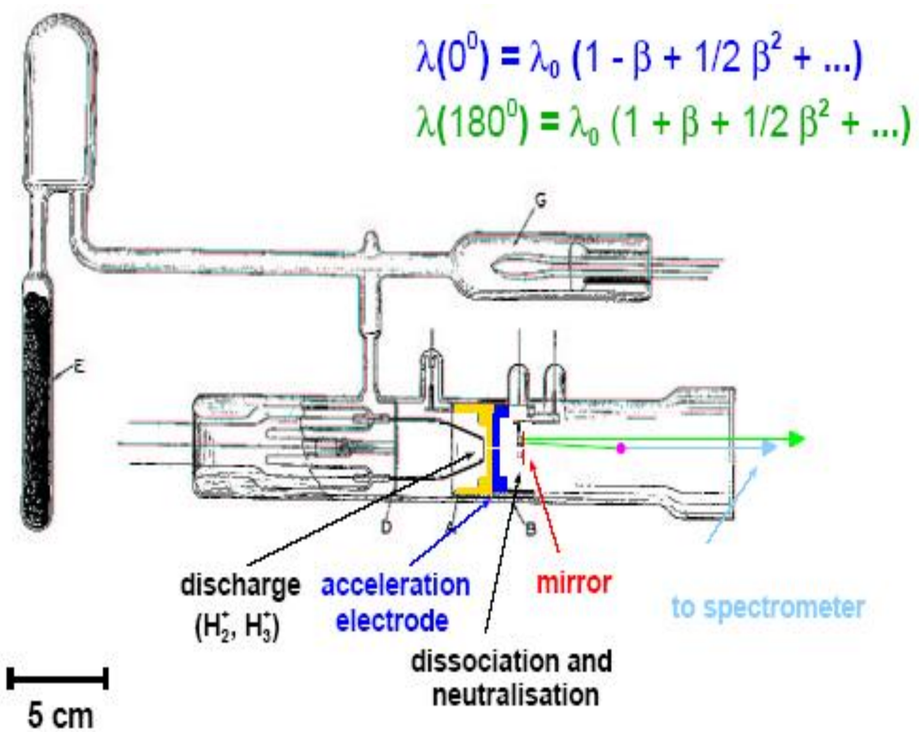
$$\Longrightarrow \nu_{\mathrm{p}}\nu_{\mathrm{a}} = \nu_0^2(1 + 2\delta\hat{\alpha}(\beta^2 + 2\vec{\beta}\vec{w}))$$

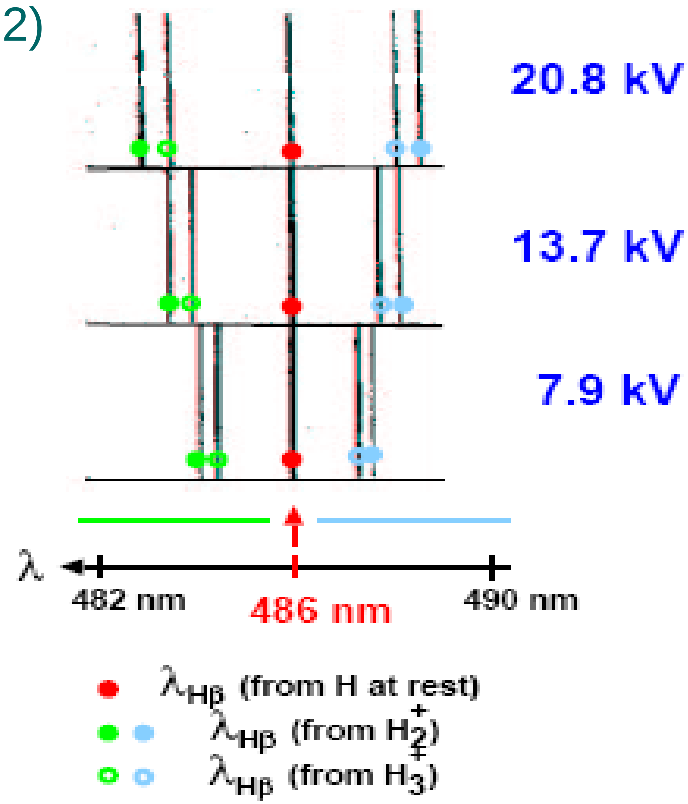
The original Ives-Stilwell experiment

Moving source: Hydrogen (H) produced in canal ray tube (β=0.0005 , (γ -1)<$1\cdot10^{-5}$)
Observer: High resolution grating spectrometer

Observing the H_β line of the Balmer series (n=4→n=2)

$\lambda\,\nu$=c in all frames →$\lambda(\theta)=\lambda_0\gamma(1-\beta\cos\theta)$





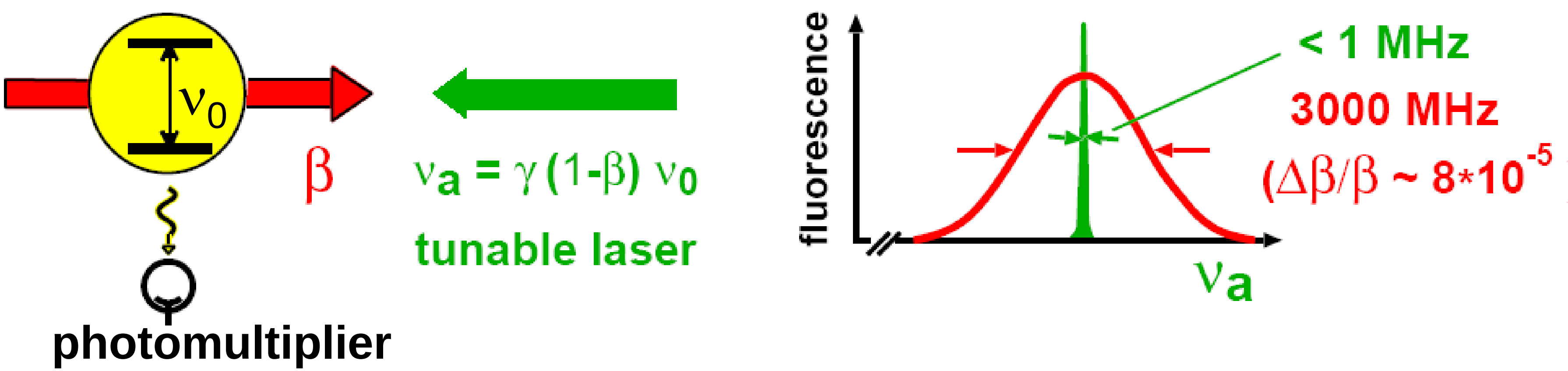
$\delta\alpha<1\cdot 10^{-2}$

Modern Ives-Stilwell Experiment

Improvements by:
- faster particles → accelerator, ion storage ring
- higher accuracy → laser spectroscopy

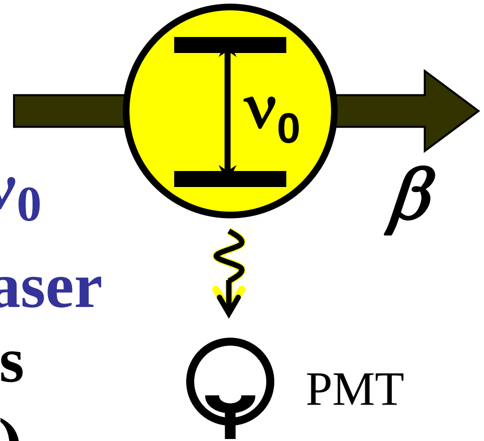
Source: frequency-stabilized lasers ($\Delta\nu/\nu < 10^{-9}$)

Moving observer: fast ion with narrow resonance ($\Delta\nu_0/\nu_0 < 10^{-8}$)



Frequency measurement now limited by Doppler broadening

Doppler-free Saturation Spectroscopy



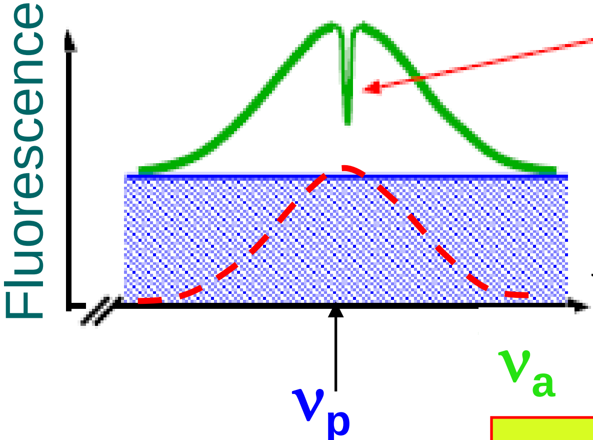
$\nu_p = \gamma\,(1 + \beta_0)\,\nu_0$

fixed frequency laser

(marking ions of velocity β_0)

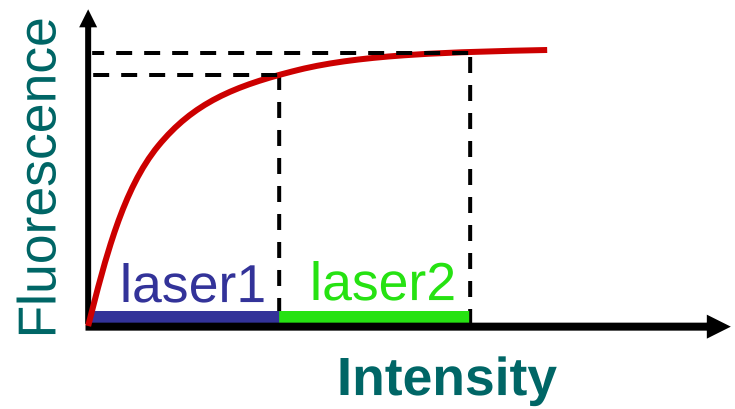
$\nu_a = \gamma\,(1 - \beta)\,\nu_0$

tunable laser



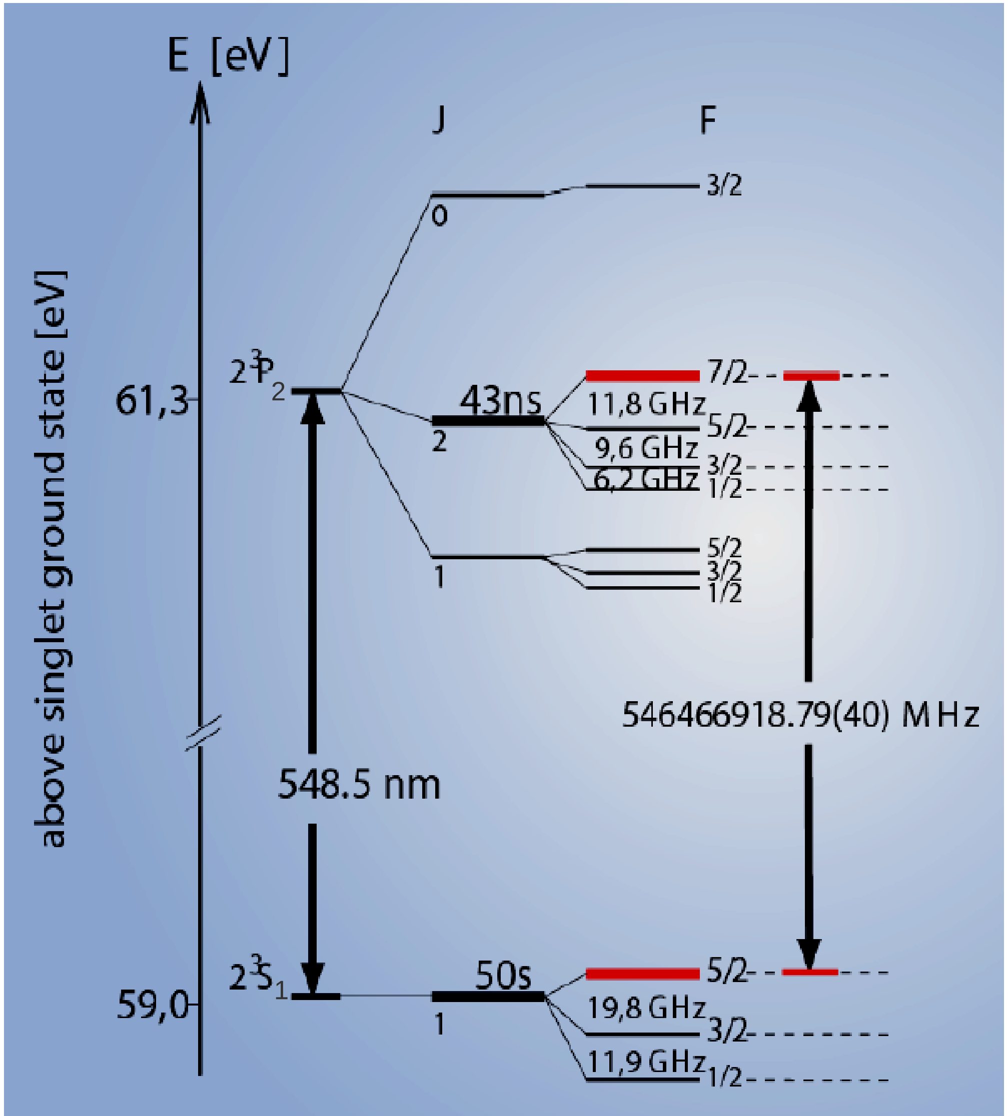
Lamb-dip

(Both laser are in resonance with the same velocity group β_0)

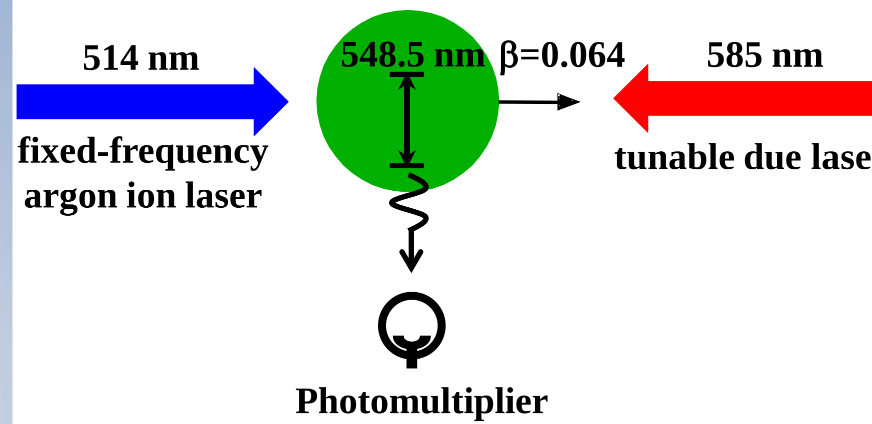


Lamb dip: $\nu_p \nu_a = \nu_0^2$

Metastable $^7Li^+$: A Suitable Candidate



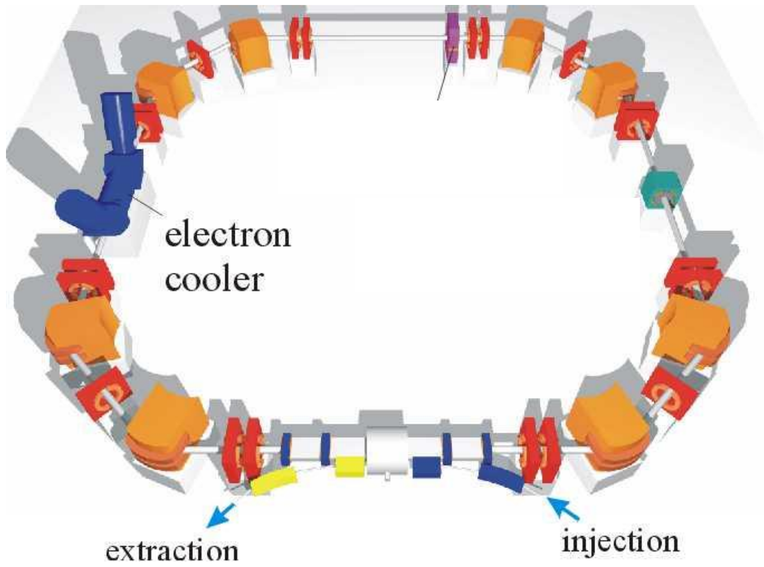
Doppler Shift of 30 000 GHz



γ=1.002

Issues:
- only 10-20% metastables in beam
- decay of metastable fraction: 8-16s in storage ring
- natural linewidth: 4 MHz

The Heidelberg Test Storage Ring (TSR)

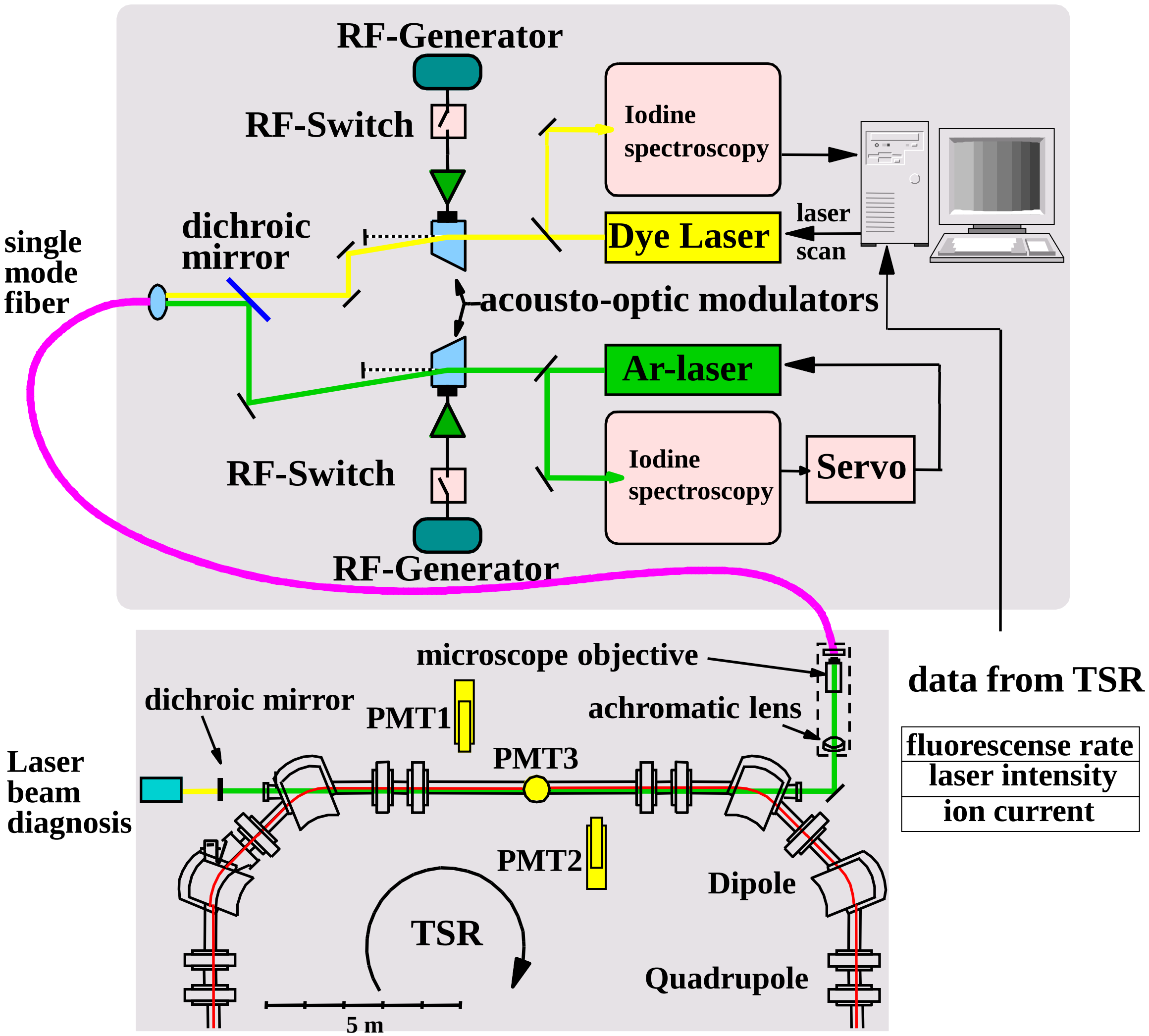


- Circumference: 55 m
- Vacuum: 5×10^{-11} mbar

$^7Li^+$-beam:

- ion number: 10^7
- velocity: v=0.064c (13.3 MeV)
- $\rightarrow \gamma = 1.002$
- diameter: 500 μm
- divergence: 50 μrad

Experimental Setup

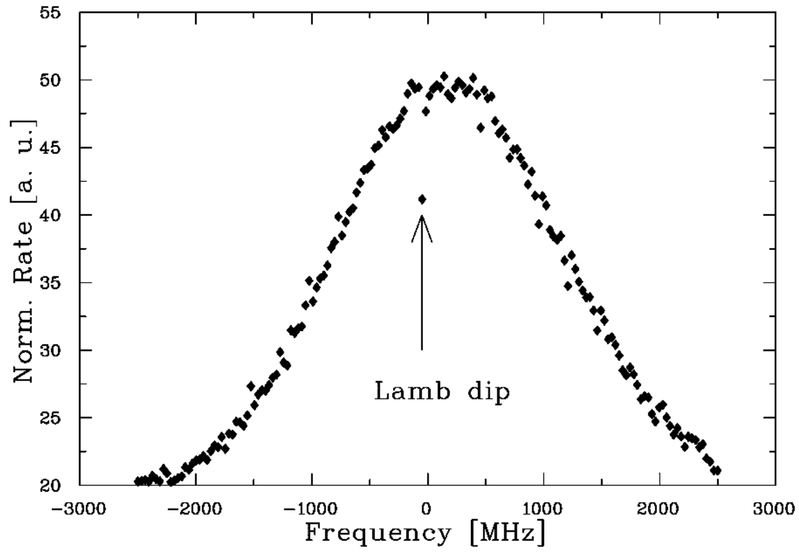


Iodine spectroscopy:
saturation spectroscopy

Lasers are sent through AOMs:

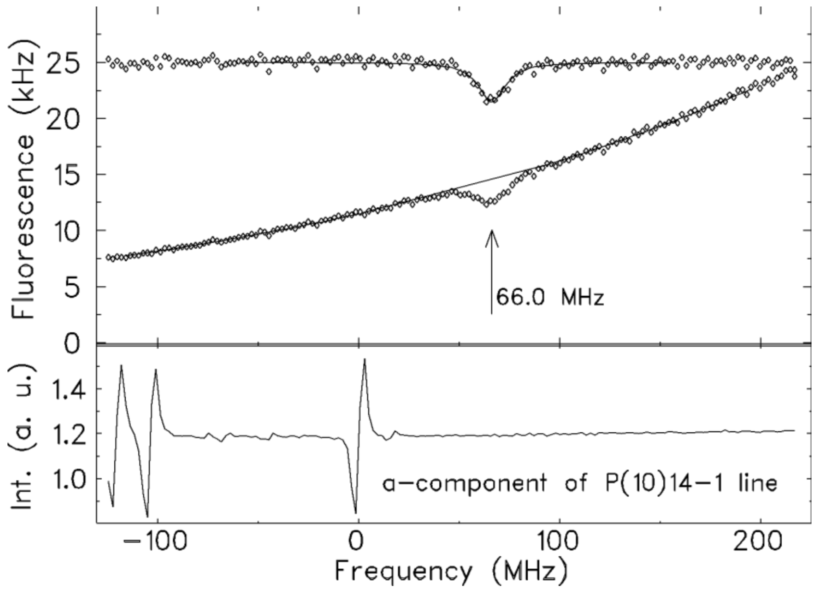
- Iodine line can be frequency-shifted with respect to Li-signal
- Lasers can be switched on and off at a high switching frequency

First Lamb Dip Measurements



•momentum spread: $\Delta p/p = 8 \times 10^{-5}$
$\rightarrow$ Doppler-width: $\Delta\nu$ = 2.8 GHz

•Lamb-dip: $\Delta\nu \approx$ 15 MHz

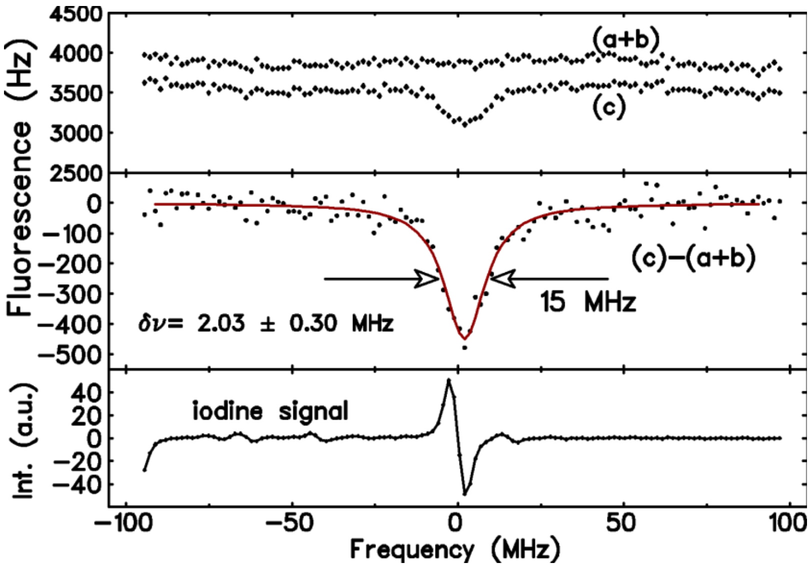


A closer look:

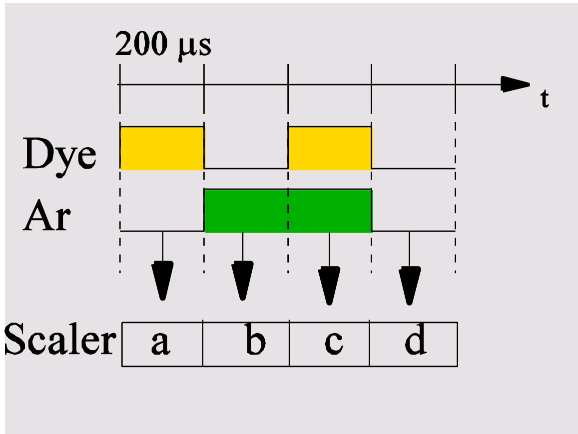
• closest iodine line 66 MHz lower in frequency than Lamb dip
• lifetime of metastables about 13 s. Exp. decrease of fluorescence

Modified Measurement Scheme

Typical run containing 80 Scans



- **AOM frequency adjusted to shift iodine line close to the Lamb dip**
- **laser scan cycle (200 data points) decoupled from ion injection cycle (46 data points) →ion beam decay largely averages out**

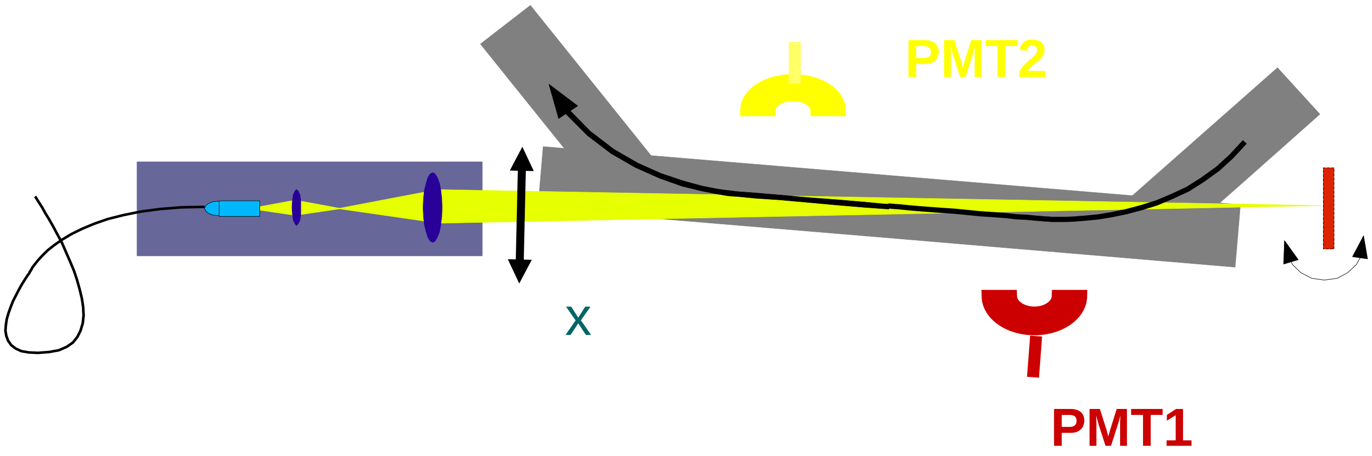


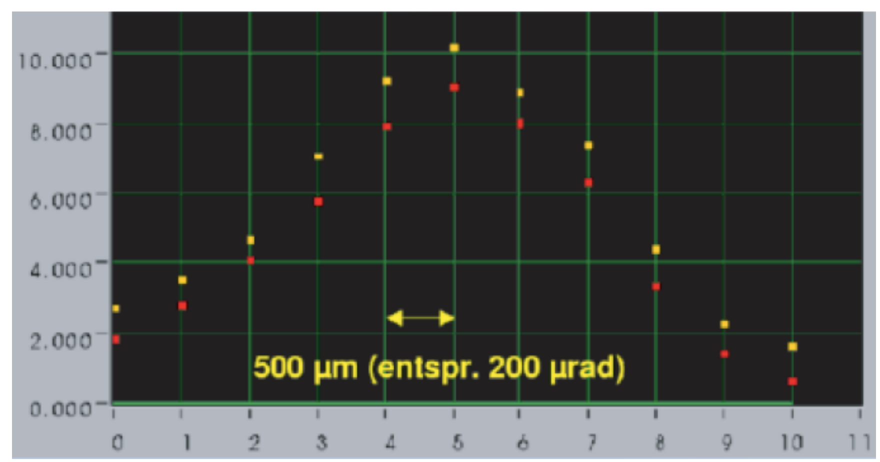
(a+b): lasers are applied separately →Doppler background only

(c): lasers applied simultaneously →Doppler background + Lamb dip

difference signal contains pure Lamb dip

Adjustment of the laser-ion beam angle

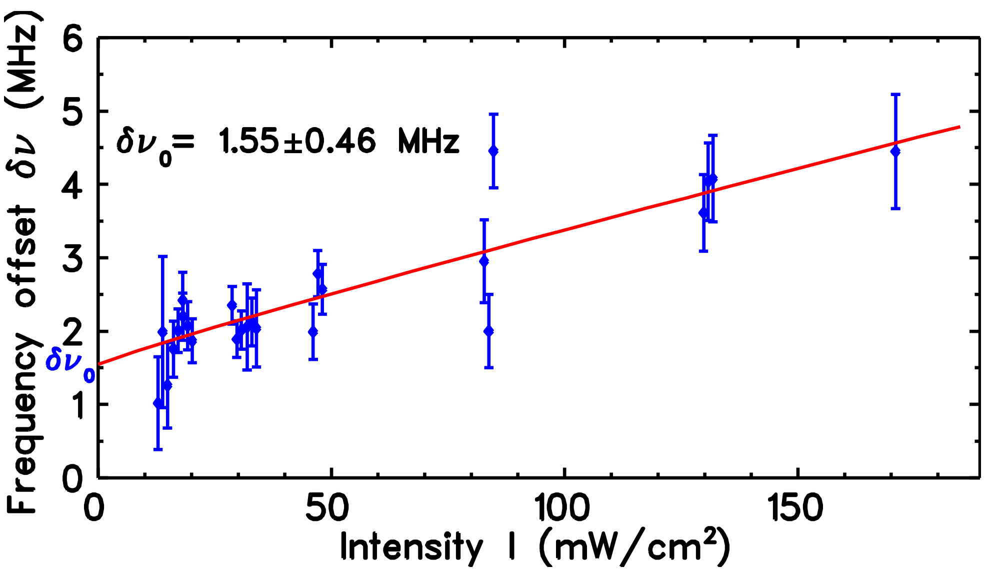




with new optical method: $\Delta\theta < 70\mu\text{rad}$

Influence of Laser Forces

The Lamb dip frequency is slightly dependent on the laser power



Reason:

laser forces locally change velocity distribution → modified Doppler background & ac-stark shift

Extrapolation to zero intensity by fitting

$$\delta\nu = \delta\nu_0 + mI^{\kappa}$$

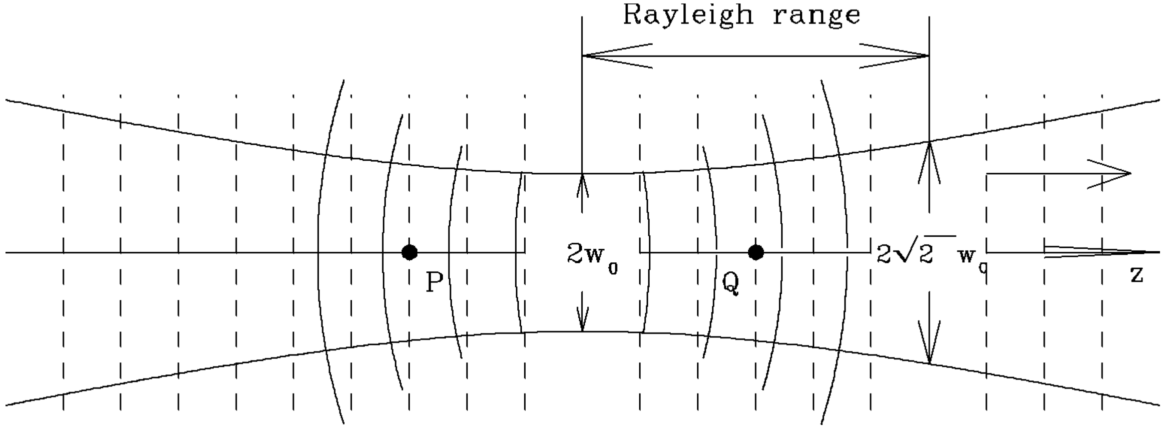
Result:

almost linear dependence (κ=0.93)

$$\delta\nu_0 = 1.55 \pm 0.46 \text{ MHz}$$

Laser Curvature Effects

Lasers generate Gaussian Beams



Phase on optical axis: $\phi(r=0,z) = \underbrace{2\pi\nu_L - k_L z}_{\text{plane wave}} + \underbrace{\arctan\frac{z}{z_R}}_{\text{Guoy Phase}}$

Frequency from the ions point of view:

$$\nu = \frac{1}{2\pi}\frac{d\varphi}{dt} = \frac{1}{2\pi}\left(\frac{\partial\varphi}{\partial t} + v\cdot\frac{\partial\varphi}{\partial z}\right)$$

$$= \nu_L + \beta\nu_L\left(-1 + \frac{c}{2\pi\nu_L}\frac{z_R}{z^2+z_R^2}\right)$$

argon laser: $\delta\nu^{\mathrm{wf}} = -665 \pm 160$ kHz

dye laser: $\delta\nu^{\mathrm{wf}} = +179 \pm 70$ kHz

Experimental Result

	Frequency (kHz)	1σ uncertainty (kHz)
iodine reference line	512 671 028 023	152
AOM shift (dye laser)	414 000	negl.
Lamb dip offset to ref.	1 550	460
Wavefront corr. (on-axis)	-665	160
laser-laser angle		40
laser-ion angle		10
ion beam divergence		50
rel. frequency calibration		50
total ν_a^{exp}	512 671 442 908	516

Comparison to Prediction from SR

- $\nu_a^{exp} = 512\ 671\ 442\ 908 \pm 516\ \text{kHz}$
- $\nu_a^{SR} = \frac{\nu_0^2}{\nu_p}$ $\qquad \nu_p = \nu_{a3-comp} + \delta\nu_{Ar}^{AOM} + \delta\nu_{Ar}^{wf}$

	Frequency (kHz)	1σ uncertainty (kHz)
iodine reference line	582 490 603 380	162
AOM shift (argon laser)	-400 000	negl.
Wavefront corr.	179	66
$^7Li^+$ rest frequency	546 466 918 790	400 (PRA 19, 1994)
total ν_a^{SR}	**512 671 443 249**	**766**

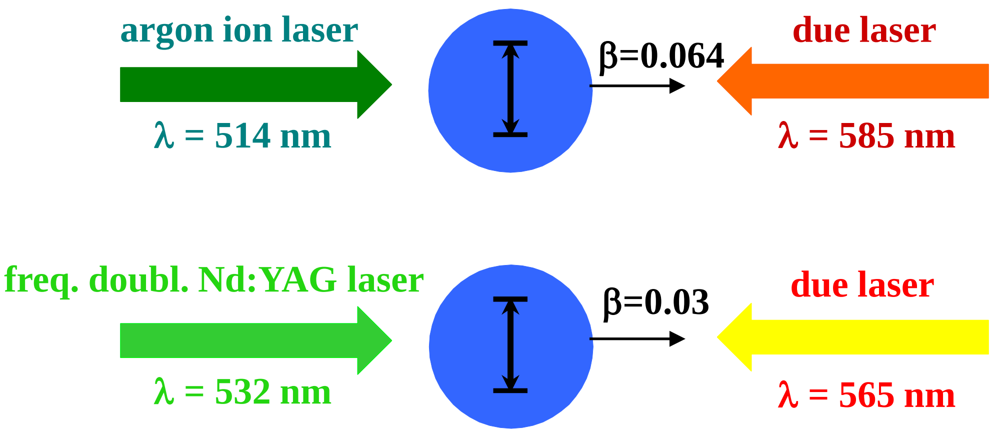
$$\delta = \nu_a^{exp} - \nu_a^{SR} = -332 \pm 924\ \text{kHz}$$

$$\boxed{\delta\hat{\alpha} < 2.2 \times 10^{-7}}$$

[Saathoff et al., PRL 19,190403 (2003)]

Measurement at low ion velocity

- The experiment is limited by the knowledge of the rest frame frequency ν_0
- Replace measurement of ν_0 by measurement at low ion velocity



$$\frac{\nu_p^h \nu_a^h}{\nu_0^2} = 1 + 2\delta\hat{\alpha}\beta_h^2$$

$$\frac{\nu_p^l \nu_a^l}{\nu_0^2} = 1 + 2\delta\hat{\alpha}\beta_l^2$$

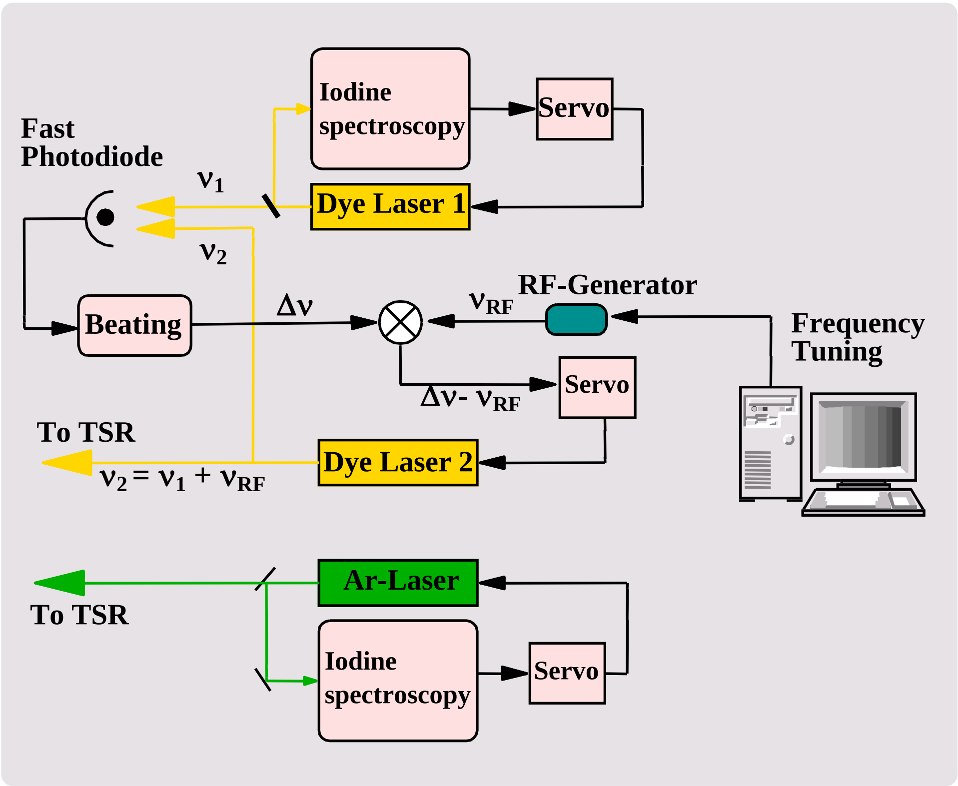
$$\frac{\nu_p^h \nu_a^h}{\nu_p^l \nu_a^l} = 1 + 2\delta\hat{\alpha}\beta_h^2\left(1 - \frac{\beta_l^2}{\beta_h^2}\right)$$

$\Delta\nu_p^{(l)}$ and $\Delta\nu_a^{(l)}$ are in the 150 kHz range

An iodine line as reference at 565 nm has been calibrated using a self-referenced frequency comb (collaboration with T. W. Hänsch, R. Holzwarth, T. Udem, M. Zimmermann, MPQ)

New Laser Setup

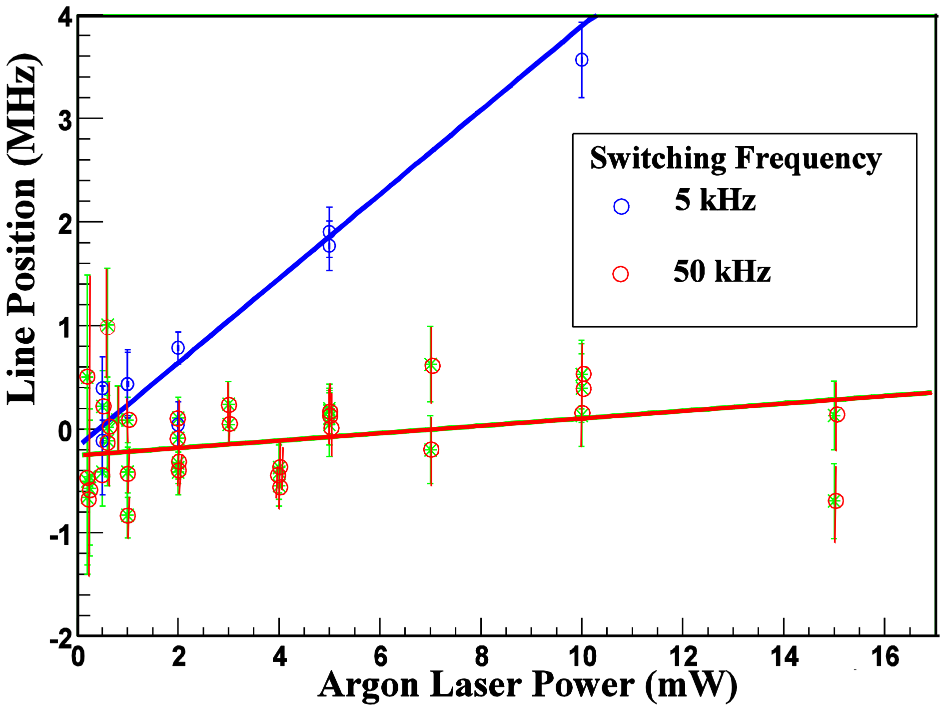
- Dye laser 1 locked to iodine
- Dye laser 2 tuned via frequency offset lock to dye laser 1



<u>Advantages</u>:

- Laser frequency calibrated at each data point
- Laser frequency can be adjusted reproducibly and in an arbitrary order to each value of the tuning range
- Stochastic scans allow for reduction of long-term effects of the laser forces

Faster Laser Switching



Intensity dependence strongly decreases with increasing switching frequency

Extrapolated frequencies from both measurements agree within the error

→Fit uncertainty is in the 100 kHz range

Experimental Results

	Frequency, [kHz]	
	β = 0.03	β = 0.064
ν_a	530 222 086 393 (145)	512 671 301 659 (145)
ν_p	563 209 456 521 (96)	582 490 363 805 (125)

$\Delta\nu/\nu = 2 \times 10^{-10}$

Calculation of $|\delta\hat{\alpha}|$ and ν_0

$$|\delta\hat{\alpha}| \leq 8.0 \cdot 10^{-8}$$

ν_0 = 546 466 918 615 (116) kHz

ν_0 = 546 466 918 790 (400) kHz
(PRA 19, 1994)

Next Steps

In the future:

• measurement at considerably higher ion velocity (β=0.34)

Standard Model Extension:

- our experiment gives an upper limit for one parameter in the photon sector:
  $\tilde{\kappa}_{tr} < 10^{-5}$ (only quoted limit to date), M.Tobar *et al.* PRD **71**, 025004 (2005)
- an analysis in the fermion sector is currently underway (C. Lane).

Summary

Most accurate limit for deviations from time dilation:

$$|\delta\hat{\alpha}| \leq 8.0 \cdot 10^{-8}$$

20fold improvement compared to non-storage ring experiments!

Future:

v=0.34*c* at GSI →push $|\delta\hat{\alpha}|$ into 10^{-9} range