THE PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 129, No. 6 — 15 MARCH 1963

Measurement of the Transverse Doppler Effect in an Accelerated System*

WALTER KÜNDIG†

Department of Physics, Purdue University, Lafayette, Indiana

(Received 15 October 1962; revised manuscript received 4 December 1962)

Using an ultracentrifuge rotor, the shift of the 14.4-keV Mössbauer absorption line of Fe^{57} in a rotating system was measured as a function of the angular velocity ω. An Fe^{57} absorber was placed at a radius of 9.3 cm from the axis of the rotor. A Co^{57} source was mounted on a piezoelectric transducer at the center of the rotor. By applying a triangularly varying voltage to the transducer, the source could be moved relative to the absorber. This arrangement makes possible the observation of the entire resonance line at various values of ω. The measured transverse Doppler shift agrees within an experimental error of 1.1% with the predictions of the theory of relativity. Possible sources of systematic errors are discussed.

INTRODUCTION

THE extremely high precision of frequency measurements, possible by the Mössbauer effect, allows to check one consequence of the theory of relativity, the "transverse Doppler effect." The basic idea of the experiment is the following: A Mössbauer source is placed in the center of a system rotating with the angular velocity ω, an absorber is mounted at a radius R_A, and a counter is at rest beyond the absorber. The γ-ray transmission through the absorber is observed as a function of the speed $R_A\omega$ of the absorber.

As was pointed out by Sherwin,[1] this experiment differs from measurements of the transverse Doppler effect for uniform translations, i.e., the observation of the second order Doppler shift in canal rays[2,3] and the measurements of the lifetime of mesons decaying in flight[4-6] (in these measurements the transverse Doppler effect was checked to an accuracy of about 10%). The frequency shift in a rotating system might be described as the transverse Doppler effect for accelerated systems, also known as the "clock paradox."[1]

When the experiment is analyzed in the inertial frame of the source, the result follows from the time dilatation in the special theory of relativity.[7] Since the relative velocity $v=\beta c$ $(\beta\ll 1)$ of source and absorber is always in a direction perpendicular to the line joining them, there exists a transverse Doppler effect giving in first approximation a fractional energy change,

$$(E_A-E_S)/E_S=(1-\beta^2)^{1/2}-1\simeq-\tfrac{1}{2}\beta^2=-R_A{}^2\omega^2/2c^2, \quad (1)$$

where E_A and E_S are the characteristic energies of the absorber and the source. However, when the experiment is analyzed in a reference frame K attached to the accelerated absorber, the problem could be treated[7] by the principle of equivalence and the general theory of relativity. The centrifugal force acting on the absorber is then interpreted as a gravitational force with the potential

$$\Phi=-\tfrac{1}{2}R_A{}^2\omega^2. \quad (2)$$

Thus, the observer in K will come to the conclusion that his clock is slowed down by the gravitational potential. The frequency ν_A measured in his frame of reference is given to a first approximation by

$$\nu_A=\nu_S(1+2\Phi/c^2)^{1/2}\simeq\nu_S(1+\Phi/c^2). \quad (3)$$

---

* This work was supported by the U. S. Atomic Energy Commission.

† Present address: Department of Physics, University of California, Los Angeles, California.

[1] C. W. Sherwin, Phys. Rev. **120**, 17 (1960).
[2] H. E. Ives and G. R. Stilwell, J. Opt. Soc. Am. **28**, 215 (1938).
[3] G. Otting, Physik. Z. **40**, 681 (1939).
[4] R. Durbin, H. H. Loar, and W. W. Harens, Phys. Rev. **88**, 179 (1952).
[5] L. M. Lederman, E. T. Booth, H. Byfield, and J. Kessler, Phys. Rev. **83**, 685 (1951).
[6] H. C. Burrowes, D. O. Caldwell, D. H. Frisch, D. A. Hill, D. M. Ritson, and R. A. Schluter, Phys. Rev. Letters **2**, 117 (1959).
[7] W. Pauli, *Theory of Relativity* (Pergamon Press, London, 1958), pp. 19, 151.

The fractional energy shift is, as before,

$$(E_A-E_S)/E_S \simeq -R_A{}^2\omega^2/2c^2. \qquad (4)$$

We thus see that the transverse Doppler effect and the time dilatation produced by gravitation appears as two different modes of expressing the same fact, namely that the clock which experiences acceleration is retarded compared to the clock at rest.

The experiment was first done by Hay, Schiffer, Cranshaw, and Egelstaff.[8] They observed the increase of the γ-ray transmission through the absorber as the velocity of the absorber increased, indicating a shift in the characteristic energy of the absorber toward lower frequency. Assuming the experimentally known line shape for the absorber at rest stayed constant, the magnitude of the frequency shift was determined. They found that within the experimental errors the effect is in agreement with the expectation.

With an improved technique it was tried to increase the precision of this fundamental experiment in relativity. Using an ultracentrifuge rotor the shift of the 14.4-keV Mössbauer absorption line of Fe^{57} was measured as a function of ω. A Co^{57} source was placed on a piezoelectric transducer at the center of the rotor, and the iron absorber at a radius R_A. By applying a triangularly varying voltage to the transducer, the source could be moved relative to the absorber. This arrangement makes possible the observation of the entire resonance line at various values of ω. Thus, in contrast to earlier measurements, the evaluation is independent of the line shape. The motion of the source caused by the transducer was calibrated in an additional experiment.

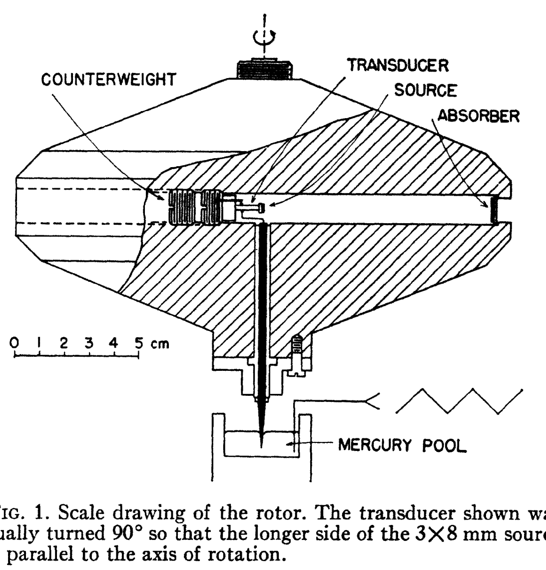

FIG. 1. Scale drawing of the rotor. The transducer shown was actually turned 90° so that the longer side of the 3×8 mm source was parallel to the axis of rotation.

[8] H. J. Hay, J. P. Schiffer, T. E. Cranshaw, and P. A. Egelstaff, Phys. Rev. Letters 4, 165 (1960); in *Proceedings of the Second Conference on the Mössbauer Effect*, edited by A. Schoen and D. M. T. Compton (John Wiley & Sons, Inc., New York, 1962).

EXPERIMENTAL ARRANGEMENT

Figure 1 shows a cross section of the rotor. It was machined from a special aluminum alloy and had a diameter of 20 cm. The source and absorber were mounted in a hole of 1-cm diam which was drilled diametrically through the rotor. The absorber, a 0.25-mil-thick foil of 91% enriched Fe^{57} was placed inside a $\frac{1}{16}$-in.-thick Plexiglas disk and was mounted at a radius $R_A=9.3$ cm. The source, about 10 mC Co^{57}, plated on a 3 mm×8 mm iron foil was glued to an isolating piece of Plexiglas mounted on the face of a piezoelectric transducer in the center of the rotor. For the transducer a plate of 10-mm length, 8-mm width and 1-mm thickness of PZT-4, a ferroelectric ceramic, was used.[9] A triangular voltage was fed to the transducer by an isolated stainless steel needle dipping into an oil-covered mercury pool.

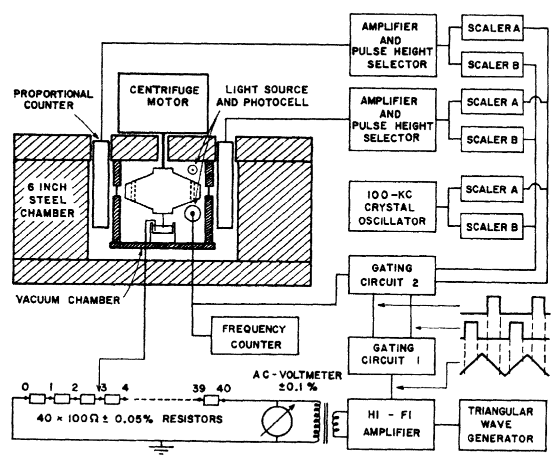

FIG. 2. Schematic diagram of the experimental arrangement. The photocell and the light source are actually turned 90° with respect to the position of the proportional counters. The two axial holes shown in the rotor are used to activate the photocell, which provides the gating pulse for the operation of the scalers.

The over-all experimental arrangement is shown in the block diagram of Fig. 2. The rotor was suspended by a flexible steel shaft, and was driven over this shaft by a Spinco ultracentrifuge motor. This suspension made the rotor self-balancing. The speed of the rotor was regulated manually to 0.1%. The rotor was spinning in an evacuated chamber to minimize frictional heating. Physical protection was provided by a 6 in. steel shielding around the vacuum chamber. The triangular wave from a Hewlett-Packard function generator of 1000.0 ± 0.5 cps was amplified to an amplitude of 100 V and kept constant to 0.2%. The voltage was then divided by 40 induction-free 100Ω resistors and applied to the transducer. The stationary counter beyond the rotor saw the diametrical hole with the absorber during about 2% of the time. In order to

[9] Manufactured by Clevite Corporation, Cleveland, Ohio.

increase this low duty cycle, two krypton-filled proportional counters were used.

The pulses of each of the two proportional counters were amplified and fed to a pulse-height analyzer selecting the 14.4-keV Mössbauer radiation. The output of the pulse-height selector was connected with two scalers A and B as indicated in Fig. 2. Scalers A and B were gated according to the motion of the source in the following way: Scalers A were counting when the voltage applied to the transducer was increasing from 20 to 80% and scalers B when the voltage was decreasing from 80 to 20%. To reduce the background additional gating was provided by a photocell so that the scalers only accepted pulses when source, absorber and counter were in a straight line. Two scalers connected with a 100 kc/sec crystal oscillator and gated also by the photocell and the triangular wave determined the gating times of scalers A and B. These gating times were used for the normalization of the counting rates. The measurements were done under arbitrarily chosen speeds of the rotor, but so that no resonance with the 1 kc/sec triangular wave occurred. Figure 3 shows an example of the measured resonance line at three different velocities of the rotor. The measured counting rates were fitted to a Lorentz curve by the least squares method, normalized so that the counting rate for $v=\infty$ was 10.

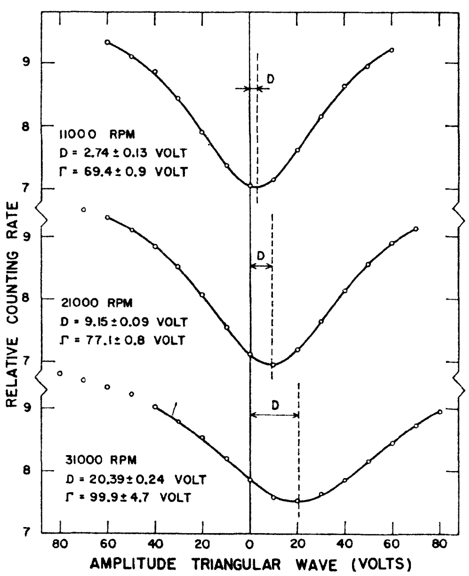

FIG. 3. Typical resonance curves. The amplitude of the triangular wave is proportional to the linear velocity of the source with respect to the absorber. The left (right) side of the plot corresponds to a motion of the source toward (away from) the absorber. The plotted curves are the fitted Lorentz curves normalized to 10 at $v=\infty$. Γ is the full width of the resonance line. With increasing speed of the rotor a considerable broadening of the resonance line was observed. The statistical errors of the points are smaller than the circles.

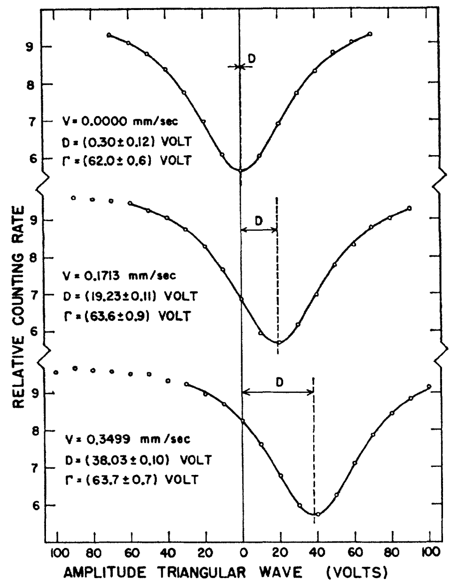

FIG. 4. Typical resonance curves measured with the linear drive. The source was moving with the indicated velocity v toward the absorber causing a linear Doppler shift D. Amplitude of the triangular voltage at the left side of the figure means the superimposed velocity of the source caused by the transducer has the same direction as v; at the right, the opposite direction. The absorption line measured at $v=0$ shows the existence of a small chemical shift of the resonance line. The plotted curves are the Lorentzians obtained by the least-squares fit method.

The motion of the source mounted on the transducer must be calibrated as a function of the amplitude of the triangular voltage applied to the piezoelectric transducer. This was performed in the following way: The source mounted on the transducer was moved toward the absorber with a constant velocity v by means of a mechanical linear drive. The mechanical drive consisted of a micrometer screw driven by a synchronous motor. The average velocity was constant to 0.1% and could be determined by measuring the time for 24 revolutions of the micrometer (=12 mm) with a photocell. The linear shift caused by moving the source toward and away from the absorber was measured as in the centrifuge experiment by applying different triangular voltages to the transducer. This method of calibration has the advantage over the use of the piezoelectric constant that it was not necessary to know the absolute amplitude and distortion of the triangular wave and the exact dimensions of the transducer. Also by using the same source and absorber as in the centrifuge experiment a possible chemical shift of the resonance line would cancel. Figure 4 shows an example of these measurements which were done every few days. The position D of the resonance line in terms of the triangular wave amplitude was determined as in the centrifuge experiment by fitting the experimental points to a Lorentz curve by the least-squares method. In Fig. 5, D is plotted as a function of the velocity of the linear drive. The points were fitted by the method

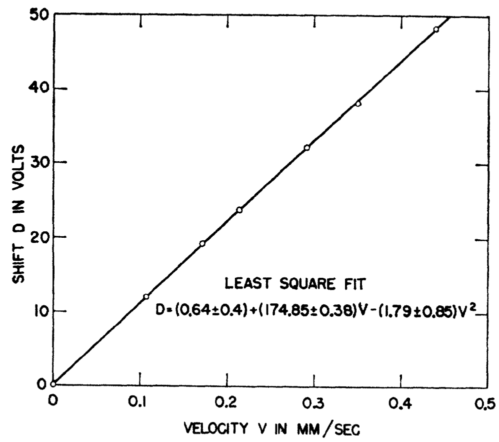

FIG. 5. Least-squares fit parabola of calibration points for the transducer. Plotted is the shift D vs the linear velocity v of the mechanical drive. In first approximation the source velocity is proportional to the triangular voltage applied to the transducer. The statistical errors are smaller than the circles.

of least squares to a parabola, e.g., $D=(0.64\pm0.40)+(174.85\pm0.38)v-(1.79\pm0.85)v^2$ where D is in volts and v in mm/sec. The calibration measurements were checked for consistency by three different linear drives, one hydraulic and two employing micrometers. It was found that within statistical errors of 0.25% the different mechanical drives gave the same result for the voltage-to-displacement ratio of the transducer.

EVALUATION AND RESULT

The calibration made it possible to calculate the shift measured in the centrifuge experiment in units of velocity of a corresponding linear Doppler shift. The velocity of the absorber $R_A\omega$ was corrected for the stretching of the rotor and the deformation of the Plexiglas disk holding the absorber. The stretching of the rotor was estimated from data obtained from the manufacturer of similar rotors made from the same material (Spinco Division of Beckman Company). For 40 000 rpm the stretching of the radius R_A (0 rpm) $=93.07\pm0.13$ mm is 0.31 ± 0.07 mm. The deformation of the Plexiglas disk was measured using compressed air with pressure corresponding to the centrifugal force at the different velocities of the rotor. It was found that, e.g., for 40 000 rpm the deformation is 0.04 ± 0.01 mm. The result was further corrected for the position of the source which, because of its finite width of 3 mm moved at a mean radius R_S of about 1 mm. The average time dilatation caused by its velocity is $R_S^2\omega^2/2c^2=0.01\pm0.005\%$ of the time dilatation $R_A^2\omega^2/2c^2$ caused by the velocity of the absorber.

The following influences were investigated and found to be negligible:

(a) Pressure effect on the absorber.[10] For the thin absorber which was held by its centrifugal force inside the Plexiglas disk this effect is less than 10^{-5} of the measured transverse Doppler effect.

(b) Temperature difference between source and absorber.[11] For example, 1°C temperature difference would give 0.45% error at 30 000 rpm. At higher velocities a slight rise in temperature of the rotor due to air friction (10–20°C at 35 000 rpm after 30–70 h run) was observed. The heat dissipation in the transducer due to the applied triangular voltage which might cause a temperature rise of the source was always less than 1 mW. This effect would have been the same in the calibrations runs as in the centrifuge experiment and would, therefore, cancel. We have no reason to assume any important temperature gradient between the source and the absorber location.

(c) Temperature effect on the piezoelectric transducer. According to the specification of the ferroelectrica used, the change of the appropriate piezoelectric constant d_{31} between 0 and 100°C is less than 1%.

(d) Pressure effect on the piezoelectric transducer. The decrease of d_{31} is proportional to ω^4 and is, e.g., 0.025% at 35 000 rpm.

The results of the measurements including the errors are given in Table I. In Fig. 6 the measured shift D is plotted as a function of the velocity $R_A\omega$ of the absorber. The parabola shown is the one expected from theory. The zero point of the parabola is fixed by calibration. A least-squares fit for the measured points based on the assumption that the time dilatation is proportional to $(R_A^2-R_S^2)\omega^2$ gives

$$(E_A-E_S)/E_S=-(1.0065\pm0.011)R_A^2\omega^2/2c^2.$$

The error includes a statistical error of 0.77% resulting from the determination of the shift in the calibration and the centrifuge experiment, and a systematical error of 0.28% from the determination of R_A.

TABLE I. Transverse Doppler shift measured as a function of the velocity of the rotor. $D/[(R_A^2-R_S^2)\omega^2]$ is calculated in the last column. This value must be compared with the theoretically expected value $1/(2c)$. The errors indicated are the sums of the statistical errors obtained by fitting the points to Lorentz curves by the least-squares fit method in the centrifuge experiment, and the 5–10 times smaller errors obtained by fitting the shifts calculated from the calibration experiment to a parabola (Fig. 5). No systematical errors are included in this Table.

Speed of rotor (rmp)	Shift D (10^{-6} m/sec)	$D/(R_A^2-R_S^2)\omega^2$ (10^{-9} sec/m)
3000	-1.5 ± 1.8	-1.7 ± 2.1
11 000	$+20.8\pm1.5$	$+1.803\pm0.127$
21 000	$+71.8\pm1.2$	$+1.705\pm0.029$
25 000	$+101.4\pm1.5$	$+1.703\pm0.026$
31 000	$+151.5\pm2.3$	$+1.653\pm0.025$
35 000	$+195.0\pm2.3$	$+1.666\pm0.020$
Weighted average		$+1.679\pm0.013$
Expected result $=1/2c$		$+1.668$

[10] R. V. Pound, in *Proceedings of the Second Conference on the Mössbauer Effect,* edited by A. Schoen and D. M. T. Compton (John Wiley & Sons, Inc., New York, 1962).

[11] B. D. Josephson, Phys. Rev. Letters **4**, 341 (1960).

As Fig. 3 shows, a considerable broadening of the resonance line with increased velocity was found. This may be explained by vibrations in the rotor. In the evaluation it is assumed that the broadening of the resonance line has no influence on its position. This is true when the rotor vibrations are random with respect to the phase of the rotation of the rotor and also with respect to the phase of the transducer vibrations. An example of a phase related rotor vibration would be the forced vibration caused by a faulty bearing of the shaft connecting the rotor with the driving unit of the centrifuge. A forced phase related vibration may give a different broadening and a different shift for the two directions of rotation and different results with different rotors. At the speed of 25 000 rpm the measurements were done under both directions of rotation, and within a statistical error of 3% the least-squares fit calculations gave the same position and the same width of the resonance line. At higher velocities only one direction of rotation was used, as the other was considered unsafe. If the drive shaft had broken, a disk threaded to the top of the rotor would have fallen on a support beneath it. With one direction of rotation the rotor would then be unscrewed from the safety disk allowing the rotor to fall to the bottom of the vacuum chamber and destroy the apparatus. Two slightly different rotors were used. One of them had a machining error so that it had a statical unbalance of 20 g cm. No systematic difference between the results obtained with the two rotors was observed. The rotors could also be mounted under different angles with respect to the shaft. No evidence of any dependence of the result upon the mounting angle was observed. Furthermore, the rather thin and flexible shaft suspending the rotor (about 2-mm diam and 10 cm long) makes the transmission of a forced vibration which would be in phase with the rotation very unlikely. The only reasonable explanation of the vibrations is that they are the almost undamped characteristic vibrations of the rotor. An amplitude of these elastic vibrations of 10–15 Å would be enough to cause the broadening observed at 31 000 rpm. The discrete values of frequencies of these vibrations makes a phase relation with the rotation improbable. These characteristic vibrations could be induced by the occasional accelerating and braking actions of the motor and its gears due to slight changes in the motor speed.

In the later phases of the experiment the resonance line became irreproducibly nonsymmetric by a few percent. Since the effect was observed in the centrifuge and the calibration experiment it cannot be explained by the above discussed rotor vibrations. This effect may be an instability in the shape of the triangular wave or the gating circuits, or a change in the mounting or the linearity of the piezoelectric transducer, but the exact cause could not be located. None of these irreproducible measurements are included in the results. The results of measurements reported earlier[12] are also omitted, because the rotor used for those measurements exploded before any check and recalibration could be performed.

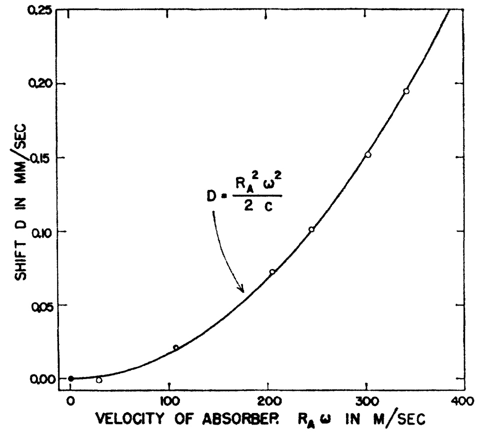

FIG. 6. Comparison of the experimental points with the theoretically expected transverse Doppler shift. The shift in units of the linear Doppler velocity is plotted against the velocity $R_A\omega$ of the absorber. The statistical error corresponds to the radius of the circles.

The result given here shows that the experimentally measured second order Doppler effect agrees within an error of 1.1% with the predictions of the theory of relativity, and is to our knowledge the most accurate determination of the transverse Doppler effect up to date.

ACKNOWLEDGMENTS

I would like to express my gratitude to Professor R. M. Steffen for valuable discussions and the hospitality afforded me during the course of this work. I would also like to thank Professor G. DePasquali of the University of Illinois for the chemical preparation of the sources.

[12] W. Kündig, Bull. Am. Phys. Soc. **7**, 350 (1962).